OPTIMAL GROUNDWATER QUANTITY MANAGEMENT FOR LAND SUBSIDENCE CONTROL

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ABSTRACT

Groundwater overpumping could cause serious land subsidence. Although groundwater management models have been widely applied to obtain optimal pumping strategies for land subsidence control, most of them have not explicitly incorporated the land subsidence into model's constraints. This study presented a groundwater management model explicitly considering land subsidence. To quantify the relation between land subsidence and drawdown, the one-dimensional consolidation equation was adopted which simultaneously accounts for the elastic and inelastic compaction due to pumping. Based on the response matrix technique along with one-dimensional consolidation equation, a groundwater quantity management model was developed which enables the determination of maximum total pumpage subject that the land subsidence do not exceed the allowable value. The developed management model was non-smooth optimization problem. To improve the solution efficiency, the non-smooth optimization was transferred into mixed integer linear programming (MILP) by introducing the binary variables. A hypothetical example was utilized to demonstrate the applicability of developed model. The results indicated that the land subsidence should be explicitly incorporated into model's constraints; otherwise the optimal total pumpage might be overestimated. Moreover, the simultaneous consideration of elastic and inelastic compaction in groundwater management was important, especially when the difference between initial head and preconsolidation head was significant.

KEY WORDS

Water resources management, Groundwater management model, Land subsidence, preconsolidation, Optimization

1. Introduction

Groundwater is an important water resource, especially for arid or semiarid regions where surface water is highly variable. Due to rapid growth in population and lack of proper management, many groundwater aquifer systems are overdeveloped resulting in serious hazards of land subsidence. Therefore, establishment of proper pumping strategies for controlling land subsidence are important aspect of groundwater quantity management.

Numerous approaches have been utilized in groundwater quantity management, among which the iterative approach is the most straightforward. In iterative approach, a specific pumping strategy is selected and simulated through the groundwater flow and land subsidence models. The model output is compared with specified design criteria. The process is repeated until a satisfactory result is determined. Onta and Gupta [1] coupled a three-dimensional groundwater flow model and a one-dimensional consolidation model to simulate the piezometric levels and land subsidence in a complex multiaquifer system of lower Central Plain of Thailand. Five pumping strategies were simulated by the model from which suitable groundwater management policies were established.

Unlike iterative approach which requires trial-and-error in finding the optimal pumping strategy, the optimization approach couples optimization algorithm with groundwater flow simulation to determine the optimal pumping strategy of aquifer system. The optimization approach has been widely applied in groundwater quantity management [2-5] and the reviews can be found in [6-7]. Based on the optimization approach, Larson et al. [8] developed a linear programming model to find the maximum total groundwater withdrawal at all production wells without causing any inelastic compaction in Los Banos-Kettleman City area of San Joaquin Valley, Calif. This was accomplished by setting the preconsolidation head as the lower bound for groundwater levels in the confined aquifer. Similar to Larson et al. [8], Phillips et al. [9] utilized the optimization approach to maximize the lowest value of head subject to the constraints that the groundwater demand was satisfied and the head did not exceed the lower bound in Lancaster, Antelope Valley, Calif. The range for the lower bound was between the initial condition and the preconsolidation head. The lower bound was set at initial condition in the first run of management model and decreased until a feasible solution was found.

Setting the preconsolidation head as the lower bound of groundwater level is a practical method to control land subsidence in optimization based groundwater quantity management. However, this method only considers the drawdown constraint, and the magnitude of land subsidence is not explicitly incorporated. Through the use of one-dimensional consolidation equation [10], Chang et al. [11] developed a mixed integer linear programming model (MILP) to determine the maximum total pumpage subject to the constraints that drawdown and land subsidence did not exceed the allowable values. The hypothetical example in Chang et al. [11] showed that joint consideration of drawdown and land subsidence was necessary. The optimal total pumpage might be overestimated and subsequently caused undesired land subsidence if only drawdown constraint was considered. Although the MILP model developed by Chang et al. [11] explicitly incorporated the land subsidence into model's constraints, the presence of preconsolidation head was ignored in the one-dimensional consolidation equation rendering the inherent assumption that the increase of drawdown due to groundwater pumping will always cause normal consolidation. Thus the occurrences of soil over-consolidation and rebound (i.e., elastic range) were neglected. Because the compaction per unit increase in drawdown in the inelastic range is considerably greater than that in the elastic range, this assumption might render an over-conservative pumping strategy, especially for the aquifer where the initial groundwater level is much higher than the preconsolidation head.

In this study, to simultaneously consider the inelastic and elastic compaction, the one-dimensional consolidation equation proposed by Tsai [10] was modified. Furthermore, through the modified one-dimensional consolidation equation, an optimal groundwater quantity management model explicitly considering land subsidence was developed. The developed management model released the limitation of inelastic compaction in Chang et al. [11] and enabled the determination of optimal pumping strategy for maximizing total pumpage or minimizing land subsidence such that the groundwater demand was satisfied and land subsidence did not exceed the allowable value.

2. Land Subsidence Simulation Model

Tsai [10] developed a one-dimensional uncoupled land subsidence model consisting of layered three-dimensional groundwater simulation and one-dimensional consolidation equation. In this study, the layered three-dimensional groundwater simulation was adopted to simulate the drawdown due to pumping, and the one-dimensional consolidation equation was modified to further consider the effect of preconsolidation head.

2.1 Description of One-dimensional Uncoupled Land Subsidence Model Developed by Tsai [10]

As the total stress is constant, the dewatering process due to pumping results in decreased pore water pressure and increased effective stress, hence leading to soil consolidation. The general three-dimensional governing equations of land subsidence for a homogeneous, isotropic and saturated soil can be stated as [12]:

$$K\left(\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}\right)=\rho_w g\left(\frac{\partial^2 U_{sx}}{\partial x \partial t}+\frac{\partial^2 U_{sy}}{\partial y \partial t}+\frac{\partial^2 U_{sz}}{\partial z \partial t}+n\gamma\frac{\partial P}{\partial t}\right) \quad (1)$$

$$\mu\nabla^2 U_{sx}+(\mu+\lambda)\frac{\partial}{\partial x}\left(\nabla\cdot\vec{U}_s\right)=\frac{\partial P}{\partial x} \quad (2)$$

$$\mu\nabla^2 U_{sy}+(\mu+\lambda)\frac{\partial}{\partial y}\left(\nabla\cdot\vec{U}_s\right)=\frac{\partial P}{\partial y} \quad (3)$$

$$\mu\nabla^2 U_{sz}+(\mu+\lambda)\frac{\partial}{\partial z}\left(\nabla\cdot\vec{U}_s\right)=\frac{\partial P}{\partial z} \quad (4)$$

where K is hydraulic conductivity; P is pore water pressure; ρ_w is density of water; g is gravitation acceleration; $\vec{U}_s$ is vector of soil displacements; U_{sx}, U_{sy}, and U_{sz} are soil displacements along the x, y, and z axes, respectively; n is porosity; t is time; μ and λ are Lame constants; and γ is fluid compression coefficient defined as $(d\rho_w / \rho_w) / dP$. Equations (1)-(4) constitute a three-dimensional coupled model that simultaneously solve for pore water pressure and soil displacement. However, its implementation to regional large scale groundwater management problems is difficult and impractical.

Tsai [10] performed an order-of-magnitude analysis on the three-dimensional coupled model and the result indicated that one-dimensional simplification (i.e., uncoupled model consisting of groundwater simulation and one-dimensional consolidation equation) is adequate when groundwater flow pattern is approximately horizontal or vertical. This approximation is plausible for regional multilayer aquifer system as groundwater flow is commonly assumed to be horizontal in aquifer and vertical in aquitard [13].

According to the result of order-of-magnitude analysis, Tsai [10] developed a one-dimensional uncoupled land subsidence model to simulate regional ground water flow and land subsidence due to groundwater pumping. For groundwater flow simulation, the one-dimensional uncoupled land subsidence model used the concept of layered three-dimensional groundwater flow in that the governing equation was integrated vertically for every layer with the assumption that the distribution of vertical pore water pressure was describable by a quadratic polynomial function. The resulting governing equation of layered three-dimensional ground water flow model can be stated as:

$$\begin{aligned}&KB\frac{\partial^2\overline{\Delta h}}{\partial x^2}+KB\frac{\partial^2\overline{\Delta h}}{\partial y^2}+\left[\frac{\partial}{\partial x}(KB)\right]\frac{\partial\overline{\Delta h}}{\partial x}+\left[\frac{\partial}{\partial y}(KB)\right]\frac{\partial\overline{\Delta h}}{\partial y}=\\&-S_s B\frac{\partial\overline{\Delta h}}{\partial t}+K\left(\frac{\partial\Delta h}{\partial z}\Big|_b\right)-\overline{S}\end{aligned} \quad (5)$$

where B is layer thickness; Δh is drawdown with positive value denoting a decrease in hydraulic head; $\overline{\Delta h}$ and $\overline{S}$ are vertical averaged drawdown and sink/source term, respectively; and S_s is specific storage.

With the assumption that (1) soil matrix is isotropic; (2) soil stress–strain relationship relating average effective stress and the average displacement follows Hooke's law of linear elasticity; and (3) displacements occur only in the vertical direction, Bear and Verruijt [14] stated that the relationship between pore water pressure change and soil vertical strain was:

$$\frac{\partial U_{sz}}{\partial z} = \frac{\Delta P}{2\mu + \lambda} \tag{6}$$

where ΔP is incremental pore water pressure. Integrating Equation (4) along the z axis, and neglecting the soil rebound due to the increase of pore water pressure, Tsai [10] obtained the one-dimensional consolidation equation as:

$$\begin{aligned} \Delta s(l,k,t) &= \frac{\rho_w g B(l,k)[\Delta h(l,k,t) - \Delta h(l,k,t-1)]}{2\mu(l,k) + \lambda(l,k)} \\ &\quad if \;\; \Delta h(l,k,t) > \Delta h(l,k,t-1) \\ \Delta s(l,k,t) &= 0 \\ &\quad if \;\; \Delta h(l,k,t) \le \Delta h(l,k,t-1) \end{aligned} \tag{7}$$

where $\Delta s(l, k, t)$ is land subsidence within layer l at point k during the tth time period; $\Delta h(l, k, t)$ and $\Delta h(l, k, t\text{-}1)$ are drawdowns of layer l, point k at the end of the tth and (t-1)th time periods, respectively.

Numerically, Tsai [10] adopted the finite analytic method to solve the layered three-dimensional groundwater flow governing equation (5), and then Equation (7) to compute land subsidence for each time period and layer. Detailed description of this model can be found in [10-11].

2.2 Modification of One-dimensional Consolidation Equation

According to Leake [15], the general relationship between land subsidence and change in drawdown could be simplified as Figure 1. In Figure 1, $\Delta h_p(l, k, t)$ is the difference between initial head and preconsolidation head at the end of the tth time period. Positive value of Δh_p denotes that initial head is higher than preconsolidation head. From Figure 1, when the drawdown is increasing from $\Delta h(l, k, t\text{-}1)$ to $\Delta h_p(l, k, t\text{-}1)$ during the tth time period (i.e., line segment AB), the soil compaction is elastic and proportional or nearly proportional to change in drawdown [16]. However, if drawdown is continuously increasing and is beyond the $\Delta h_p(l, k, t\text{-}1)$ during the tth time period, the inelastic and permanent soil compaction will occur (i.e., line segment BC). The compaction per unit increase in drawdown in the inelastic range is considerably greater (perhaps by an order of magnitude) than that in the elastic range. Unlike the elastic compaction, the relationship between inelastic compaction and change in drawdown is nonlinear, which means that the slope of segment BC is not constant, but function of effective stress. Numerous researches have linearized the relationship by assuming that inelastic compaction is proportional to change in drawdown [10, 16-17]. Leake [15] pointed out that this assumption will not produce significant error for deep sediments because the change of effective stress (or drawdown) is much less than the total stress. For shallow sediments, the inelastic compaction will be slightly overestimated. Notice that in the inelastic compaction, Δh_p is simultaneously increasing with the increase in drawdown. At the end of the tth time period, $\Delta h_p(l, k, t)$ is equal to $\Delta h(l, k, t)$.

In Figure 1, if drawdown is decreasing from $\Delta h(l, k, t)$ to $\Delta h(l, k, t+1)$ during the (t+1)th time period due to the recovery of groundwater level, the elastic soil expansion (or rebound) will occur (i.e., line segment CD). Like the elastic compaction, the elastic expansion is proportional or nearly proportional to change in drawdown. Besides, the slope of line segment CD is approximately equal to that of line segment AB with opposite sign.

From Equation (7) and Figure 1, one can find that the presence of preconsolidation head was ignored in Tsai [10] rendering the inherent assumption that the increase of drawdown due to groundwater pumping will always cause normal consolidation. Thus the occurrences of soil over consolidation and rebound were neglected. This assumption might be only applicable for the prediction of future land subsidence in a continuously over-developed aquifer system without any groundwater management for land subsidence control. To release this limitation and simultaneously consider the inelastic and elastic behavior of land subsidence, assume that (1) the land subsidence is proportional to change in drawdown; and (2) the ratio of elastic to inelastic compaction per unit increase in drawdown is α (α << 1); then the one-dimensional consolidation equation can be modified as:

$$\begin{aligned} &if \;\; \Delta h(l,k,t) \ge \Delta h_p(l,k,t-1) \Rightarrow \\ &\quad \Delta s(l,k,t) = \alpha C_c\left[\Delta h_p(l,k,t-1) - \Delta h(l,k,t-1)\right] + \\ &\qquad\qquad C_c\left[\Delta h(l,k,t) - \Delta h_p(l,k,t-1)\right] \end{aligned} \tag{8}$$

$$\begin{aligned} &if \;\; \Delta h(l,k,t) < \Delta h_p(l,k,t-1) \Rightarrow \\ &\quad \Delta s(l,k,t) = \alpha C_c\left[\Delta h(l,k,t) - \Delta h(l,k,t-1)\right] \end{aligned} \tag{9}$$

$$\Delta h_p(l,k,t) = Max[\Delta h(l,k,t), \Delta h_p(l,k,t-1)] \tag{10}$$

where $C_c = \rho_w gB/(2\mu+\lambda)$. Equation (9) is adopted if drawdown at the end of the tth time period is less than Δh_p at the end of the (t-1)th time period, otherwise the inelastic compaction will occur and Equation (8) is used instead. The first and second terms in the right hand side of Equation (8) account for the elastic and inelastic compaction, respectively. The Δh_p at the end of the tth time period will be reset to the new value based on Equation (10).

Figure 2 shows the procedures for simulating the land subsidence within layer l at point k during the tth time period. The total land subsidence at point k is:

$$\Delta s(k) = \sum_{l=1}^{L} \sum_{t=1}^{NT} \Delta s(l,k,t) \tag{11}$$

where L is number of layers and NT is number of time periods.

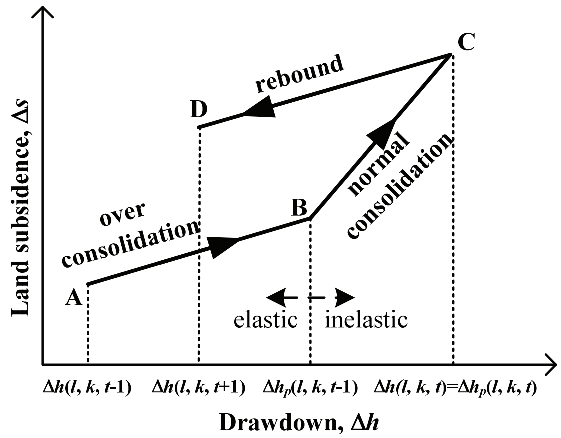

Figure 1. Relationship between land subsidence and change in drawdown.

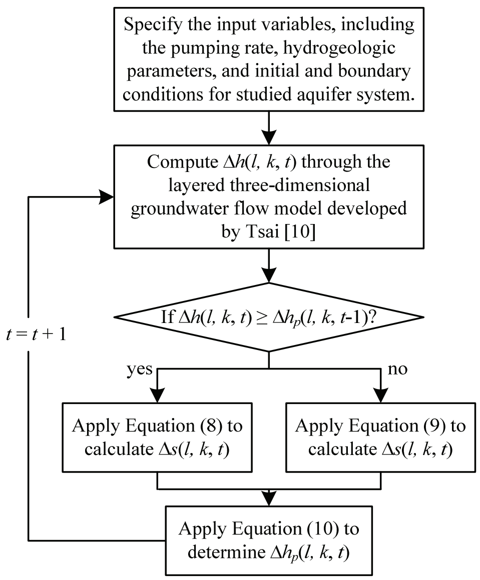

Figure 2. Procedures for simulating the drawdown and land subsidence in this study.

3. Development of Groundwater Quantity Management Model

3.1 Model Architecture

For a groundwater quantity management problem involving maximization of pumpage subject to land subsidence constraints, the management model can be formulated as:

Maximize

$$\sum_{j=1}^{NP}\sum_{i=1}^{NT} Q(j,t) \tag{12}$$

Subject to

$$\Delta s(k) = \sum_{l=1}^{L}\sum_{t=1}^{NT} \Delta s(l,k,t) \le \Delta s^*(k) \qquad k = 1,\ldots,NC \tag{13}$$

$$0 \le Q(j,t) \le Q^*(j,t) \qquad j = 1,\ldots,NP;\ t = 1,\ldots,NT \tag{14}$$

where NP is number of pumping wells; NC is number of control points; $Q(j, t)$ is pumpage at the jth pumping well during the tth time period; $\Delta s^*(k)$ is allowable land subsidence for the kth control point at the end of the NTth time period; and $Q^*(j, t)$ is allowable pumpage at the jth pumping well during the tth time period.

Equations (12) to (14) were established in the aspect of maximizing groundwater mining. In contrast, for minimizing the environmental impact due to groundwater mining, the management model can be formulated as:

Minimize

$$Max[\mathbf{\Delta s}(NC)] \tag{15}$$

Subject to

$$\Delta s(k) = \sum_{l=1}^{L}\sum_{t=1}^{NT} \Delta s(l,k,t) \le \Delta s^*(k) \qquad \forall k \tag{13}$$

$$\sum_{j=1}^{NP} Q(j,t) \ge Q^D(t) \qquad \forall t \tag{16}$$

$$0 \le Q(j,t) \le Q^*(j,t) \qquad \forall j;\ \forall t \tag{14}$$

where $\mathbf{\Delta s}(NC)$ is a NC-dimensional vector consisting of land subsidence at NC control points; and $Q^D(t)$ is groundwater demand during the tth time period. Equations (15) and (16) attempted to find the optimal pumping strategy that minimizing the highest value of land subsidence while maintained the groundwater demand.

Because the management model for minimizing environmental impact (i.e., Equations (13) – (16)) was a MiniMax optimization problem, a dummy variable Z was introduced to transfer the original model into a general optimization problem. Thus, the objective function can be rewritten as:

Minimize

$$Z \tag{17}$$

with an additional constraint as:

$$Z \ge \Delta s(k) \qquad \forall k \tag{18}$$

To solve above management models, either maximizing groundwater mining or minimizing environmental impact, an explicit equation described the relationship between Q and Δs was required. Through the response matrix technique [6], the relationship between pumpage and drawdown can be formulated as:

$$\Delta h(l,k,t) = \sum_{j=1}^{NP}\sum_{i=1}^{t} \beta(l,k,j,t-i+1)Q(j,i) \qquad \forall l;\ \forall k;\ \forall t \tag{19}$$

where β is unit response coefficient representing the drawdown of the lth layer at the kth control point at the end of the tth time period due to unit pumpage at the jth pumping well during the ith time period; and $i \le t$. From Equation (19), the difference in head between tth and (t-1)th time period is:

$$\Delta h(l,k,t) - \Delta h(l,k,t-1) = \sum_{j=1}^{NP} \sum_{i=1}^{t} [\beta(l,k,j,t-i+1) - \beta(l,k,j,t-i)] Q(j,i) \quad (20)$$

Furthermore, the one-dimensional consolidation equations (8) to (10) can be transformed into:

$$G(l,k,t) = \begin{cases} \Delta h(l,k,t) - \Delta h_p(l,k,t-1) & \text{if } \Delta h(l,k,t) - \Delta h_p(l,k,t-1) > 0 \\ 0 & \text{if } \Delta h(l,k,t) - \Delta h_p(l,k,t-1) \le 0 \end{cases} \quad (21)$$

$$\Delta s(l,k,t) = \alpha C_c [\Delta h(l,k,t) - \Delta h(l,k,t-1)] + C_c (1-\alpha) G(l,k,t) \quad (22)$$

$$\Delta h_p(l,k,t) = G(l,k,t) + \Delta h_p(l,k,t-1) \quad (23)$$

Equation (21) attempted to determine whether the compaction was inelastic or elastic during the tth time period, while Equation (22) was used to calculate land subsidence.

In summary, the management model for maximizing groundwater mining was composed of objective function (12) and constraints (13), (14), and (19) - (23). Besides, the management model for minimizing environmental impact was composed of objective function (17) and constraints (13), (14), (16) and (18) - (23).

2.2 Solution Technique

The developed management models were nonsmooth optimization problems because G was a nondifferentiable function at the origin. Although several algorithms have been developed to solve the nonsmooth optimization problem, the convergence and global optimality of the solution could not be guaranteed, especially when the problem size (in terms of the number of constraints and decision variables) was large [18-19]. To circumvent such a situation, the nonsmooth optimization problem was transformed into mixed integer linear programming (MILP) by introducing additional binary variables, $m(l, k, t)$, and Equations (21) and (23) were replaced by the following new constraints:

$$\Delta h(l,k,t) - \Delta h_p(l,k,t-1) - UP \times m(l,k,t) \ge -UP \quad \forall l;\ \forall k;\ \forall t \quad (24)$$

$$\Delta h(l,k,t) - \Delta h_p(l,k,t-1) - UP \times m(l,k,t) \le 0 \quad \forall l;\ \forall k;\ \forall t \quad (25)$$

$$\Delta h(l,k,t) - \Delta h_p(l,k,t-1) - G(l,k,t) \le 0 \quad \forall l;\ \forall k;\ \forall t \quad (26)$$

$$\Delta h(l,k,t) - \Delta h_p(l,k,t-1) - G(l,k,t) - UP \times m(l,k,t) \ge -UP \quad \forall l;\ \forall k;\ \forall t \quad (27)$$

$$G(l,k,t) - UP \times m(l,k,t) \le 0 \quad \forall l;\ \forall k;\ \forall t \quad (28)$$

$$G(l,k,t) \ge 0 \quad \forall l;\ \forall k;\ \forall t \quad (29)$$

$$\Delta h(l,k,t) - \Delta h_p(l,k,t) \le 0 \quad \forall l;\ \forall k;\ \forall t \quad (30)$$

$$\Delta h(l,k,t) - \Delta h_p(l,k,t) - UP \times m(l,k,t) \ge -UP \quad \forall l;\ \forall k;\ \forall t \quad (31)$$

$$\Delta h_p(l,k,t-1) - \Delta h_p(l,k,t) \le 0 \quad \forall l;\ \forall k;\ \forall t \quad (32)$$

$$\Delta h_p(l,k,t-1) - \Delta h_p(l,k,t) - UP \times [1 - m(l,k,t)] \ge -UP \quad \forall l;\ \forall k;\ \forall t \quad (33)$$

where UP is positive coefficient with large value; and $m(l, k, t)$ equals 0 or 1 only. For the case of $\Delta h(l, k, t) > \Delta h_p(l, k, t\text{-}1)$, from Equation (25) one can find that $m(l, k, t) = 1$ and the inelastic compaction will occur during the tth time period. Besides, Equations (26) and (27) imposed that $G(l, k, t) = \Delta h(l, k, t) - \Delta h_p(l, k, t\text{-}1)$. On the other hand, if $\Delta h(l, k, t) \le \Delta h_p(l, k, t\text{-}1)$, $m(l, k, t) = 0$ (i.e., Equation (24)) and the elastic compaction or rebound will occur during the tth time period.

In this study, the previous MILP was solved by the branch-and-bound (B&B) method [20]. The B&B is the most widely applied method in solving integer or mixed integer programming. The main concept of B&B for a MILP with binary variables is to branch the problem into the following conditions: one binary variable equals to 0 or 1, and the other binary variables were treated as continuous. And then repeat this procedure by enumerative tree until the optimal solution is found. The B&B method solves linear programming during each step of branching, therefore the optimal solution can be thought as a global one.

4. Verification of Management Model

The developed management model was verified through the trial-and-error method in a hypothetical aquifer basin. The verification procedures were presented in Figure 3.

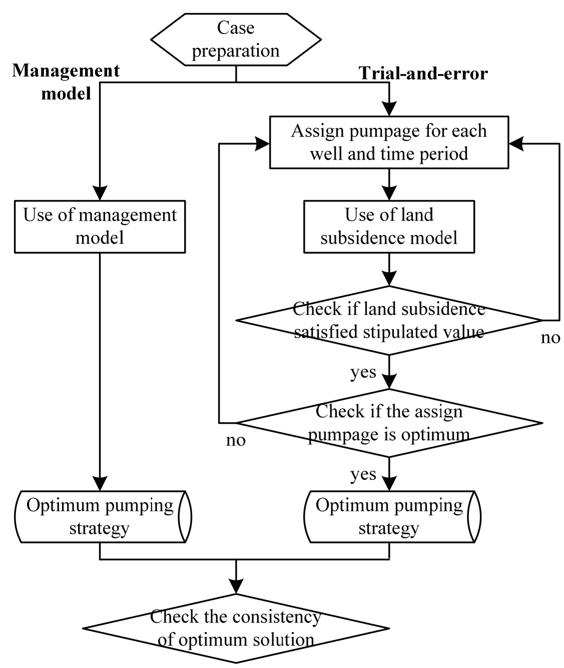

Figure 3. Flowchart of model verification.

Consider a homogeneous, isotropic confined aquifer basin with one fully penetrated pumping well as shown in Figure 4. The area and thickness of aquifer are 20 km × 16 km and 80 m, respectively. The aquifer domain is discretized into 11 × 9 nodes with equal grid space of 2 km × 2 km. The boundary conditions are no flux at the top and bottom while a constant head on the sides. The hydraulic conductivity (K) and Lame constants (μ and λ) are 2.0×10^{-4} m/s and 5.0×10^{8} N/m^2, 1.0×10^{9} N/m^2, respectively. The ratio of elastic to inelastic compaction per unit increase in drawdown is 0.1 (α = 0.1). Initially, the piezometric head is uniformly distributed and is 15 m higher than the preconsolidation head. The problem is to determine the maximum total pumpage from the production well over two time periods of six months each, such that the resulting land subsidence at all grids will not exceed 3.0 cm.

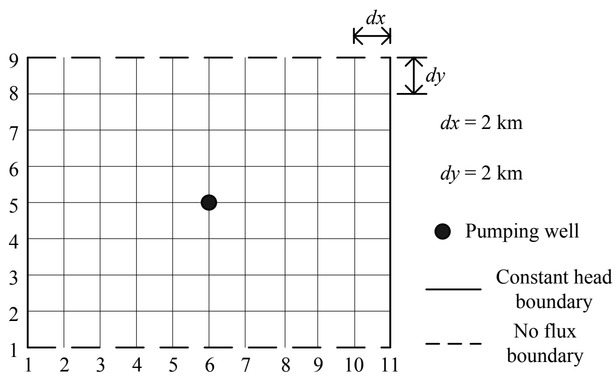

Figure 4. Hypothetical aquifer basin used in verification.

Through the use of groundwater management model, the maximum total pumpage was 4.41 m^3/s and the pumping rates during the first and second time period were 2.24 m^3/s and 2.17 m^3/s, respectively. Under the optimum pumping pattern, the maximum land subsidence at the end of the last time period was 3.0 cm which occurred at the pumping well.

Following Figure 3, a trial-and-error procedure along with numerical land subsidence simulation was performed. The results of trial-and-error were summarized in Table 1. From Table 1, the maximum total pumpage obtained from trial-and-error procedure was identical to that obtained from management model, thus the accuracy of proposed management model was verified.

Table 1. Trial-and-error results in model verification

unit: m^3/s

Pumping rate during the first period	Pumping rate during the second period	Total pumpage
1.0	2.21	3.21
2.0	2.17	4.17
2.1	2.17	4.27
2.2	2.17	4.37
2.21	2.17	4.38
2.22	2.17	4.39
2.23	2.17	4.4
2.24	**2.17**	**4.41**
2.25	2.1	4.35
2.3	1.63	3.93
3.0	N/A	N/A

N/A: no feasible solution because land subsidence exceeded 3.0 cm.

5. Application

In this example, three different groundwater management models, including (a) head constraints only (e.g., [8-9]); (b) land subsidence constraints without considering preconsolidation head (e.g., [11]); and (c) land subsidence constraints considering preconsolidation head (developed by this study), were utilized to find the optimum pumping patterns under designed criteria.

Consider a hypothetical multi-layer aquifer basin with five pumping wells (A to E) and five control points (a to e) as shown in Figure 5. The control points a, b, c, d, and e coincided with the production wells A, B, C, D, and E, respectively. The aquifer basin is divided into three zones (I, II, and III) based on their different, but isotropic, hydrogeologic parameters in each zone. The aquifer domain is discretized into 11 × 9 nodes with equal grid space of 2 km × 2 km. The multi-layer aquifer basin consists of an upper confined aquifer, an extensive confining unit (aquitard), and a lower confined aquifer. The vertical layer distribution and the corresponding parameter values of the aquifer basin are summarized in Figure 6. The ratios of elastic to inelastic compaction per unit increase in drawdown are 0.1 and 0.15 for aquifer (coarse grained) and aquitard (fine grained), respectively.

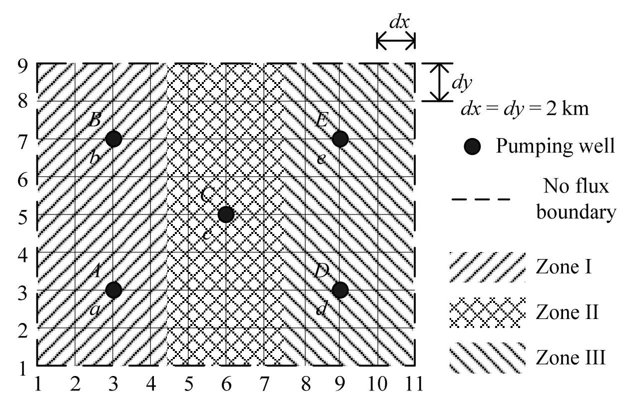

Figure 5. Horizontal view of hypothetical aquifer basin.

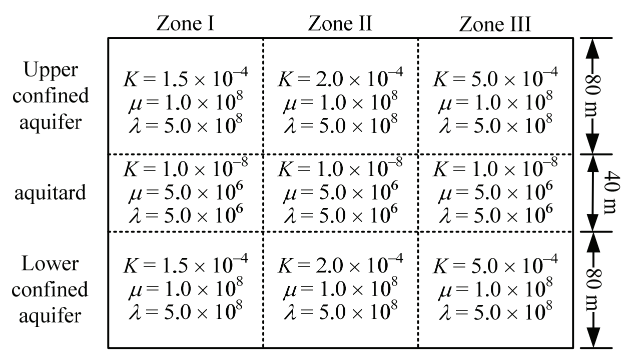

Figure 6. Vertical view and the parameter values of hypothetical aquifer basin.

Table 2. Summaries of management model inputs.

Adopted management model			(a)	(b)	(c)
Constraints	Drawdown	Zone I	$\Delta h_{p,0}$	N/A	N/A
		Zone II	$\Delta h_{p,0}$	N/A	N/A
		Zone III	$\Delta h_{p,0}$	N/A	N/A
	Land subsidence	Zone I	N/A	1 cm	1 cm
		Zone II	N/A	2 cm	2 cm
		Zone III	N/A	3 cm	3 cm
Pumping capacity	The maximum pumping capacity is 0.09 m^3/s for all production wells.				
Initial conditions	Case-1: Initial head is 5 m higher than initial preconsolidation head, $\Delta h_{p,0} = 5$ m Case-2: Initial head is 1 m higher than initial preconsolidation head, $\Delta h_{p,0} = 1$ m				
N/A: The constraints are not considered due to model limitations.					

Assume that the hypothetical aquifer basin has experienced serious land subsidence hazard due to groundwater overpumping, and the maximum annual land subsidence within Zone I, Zone II, and Zone III are 2.0 cm, 4.0 cm, and 6.0 cm, respectively. To mitigate the land subsidence hazard, the most straightforward strategy is to immediately prohibit groundwater pumping. However, this might induce huge impact on the water supply. Thus, the government agency attempts to adjust the pumping policy such that the maximum annual land subsidence could be reduced by half. According to the above statements, the problem is to determine the maximum total pumpage from the five production wells over four time periods of three months each, such that the land subsidence for Zone I, Zone II, and Zone III at the end of the last time period will not exceed 1.0 cm, 2.0 cm, and 3.0 cm, respectively. Besides, two cases involving different initial conditions were considered (see Table 2). Three different management models (a), (b), and (c) were utilized to find the maximum total pumpage However, since the model (a) only considered head constraint, the initial preconsolidation head was set as lower bound for groundwater level.

The maximum total pumpage obtained from the three management models were summarized in Table 3. The values of land subsidence listed in Tables 3 were calculated by the land subsidence simulation model (i.e., Figure 2) along with optimum pumping rates. The bolded values denote that the land subsidence exceed the maximum allowable values.

For management model (a), the land subsidence at control points *a* and *b* exceeded the maximum allowable value (1.0 cm) in Case-1. This consequence implied that in Case-1, the maximum allowable land subsidence at control points *a* and *b* lie in the elastic range. However, due to the lack of land subsidence constraints, when model (a) was searching the optimum pumping patterns, it only prevents the head from being lower than the preconsolidation head, and without actually accounting for the magnitude of land subsidence. Thus, the total pumpage obtained from model (a) was overestimated resulting in undesired land subsidence. On the other hand, in Case-2, the land subsidence at all control points were much lower than the maximum allowable values, which implied that the maximum total pumpage was underestimated. More groundwater resources could be mined while satisfied the stipulated maximum allowable land subsidence. The above discussion showed that the head constraints can avoid the occurrence of inelastic compaction; however, without land subsidence constraints, the effect of land subsidence on groundwater management cannot be explicitly considered. Thus the maximum total pumpage might be over or underestimated.

From Table 3, because management model (b) ignored the presence of preconsolidation head, the maximum total pumpage were identical in all cases. Besides, the maximum total pumpage obtained from model (b) were much lower than that obtained from model (c). This phenomenon can be explained due to the fact that model (b) assumed that positive drawdown will always cause inelastic compaction. Initially, the head was higher than the preconsolidation head in this example and the soil compaction was in elastic range, thus the land subsidence was overestimated when model (b) was searching the optimum solution. The land subsidence constraints were rapidly bound by the overestimated value resulting in an underestimated maximum total pumpage. The above discussion showed that model (b) always obtained over-conservative optimum pumping patterns because it ignored the influence of preconsolidation head. More groundwater resources could be mined while satisfied the designed criteria. The difference in maximum total pumpage between model (b) and (c) was more significant when the difference between head and preconsolidation head at initial got larger.

Table 3. Pumping rate and land subsidence under optimality

Case	Model	Maximum total pumpage	Land subsidence at control points (cm)				
			a	*b*	*c*	*d*	*e*
1	(a)	1.8 m^3/s	**1.23**	**1.23**	1.06	0.87	0.87
	(b)	0.42 m^3/s	0.14	0.14	0.30	0.34	0.34
	(c)	1.69 m^3/s	1.0	1.0	1.03	0.86	0.86
2	(a)	0.77 m^3/s	0.38	0.38	0.35	0.38	0.38
	(b)	0.42 m^3/s	0.14	0.14	0.30	0.34	0.34
	(c)	1.33 m^3/s	1.0	1.0	2.0	2.48	2.48

6. Conclusion

Groundwater quantity management might involve the design of an optimum spatial and temporal pumping strategy to meet demands, while also controlling the land subsidence. Such planned strategy can be evolved through the utilization of optimization based management models. In this study, a groundwater quantity management model simultaneously considering the inelastic and elastic compaction was developed using the response matrix technique along with the one-dimensional consolidation. The developed model enables the determination of optimal pumping strategy for maximizing total pumpage or minimizing land subsidence such that the groundwater demand is satisfied and land subsidence do not exceed the allowable value. The applicability of developed management model was demonstrated through a hypothetical example and the results showed that explicit consideration of land subsidence is necessary.

Two major concerns should be considered when applying the proposed management model in the complex real world problems. The first one is the solution efficiency of the MILP which is mostly dependent on the number of binary variables. Thus, reducing the number of time periods or control points could improve the efficiency of the MILP solver. The second concern is the applicability for the use of response matrix technique in the aquitard layer. The storage effect in the aquitard layer might result in the nonlinear relation between drawdown and pumpage. According to Bredehoeft and Pinder [21], if the nondimensional time factor $t^* = Kt/S_sB^2$ is larger than 0.5, the storage effect can be neglected. In practice, the storage effect can be reduced through the increase of time period (Δt) or dividing an aquitard layer into several virtual layers.

References

[1] P.R. Onta, and A.D. Gupta, Regional management modeling of a complex groundwater system for land subsidence control, *Water Resour. Manage.*, *9*(1), 1995, 1-25.

[2] W.W-G. Yeh, Systems analysis in ground-water planning and management, *J. Water Resour. Plan. Manage.*, *118*(3), 1992, 224-237.

[3] D.P. Ahlfeld, and M. Heidari, Application of optimal hydraulic control to ground-water systems, *J. Water Resour. Plan. Manage.*, *120*(3), 1994, 350-365.

[4] B.J. Wagner, Recent advances in simulation-optimization groundwater management modeling, *Rev. Geophys.*, *33*(S1), 1995, 1021–1028.

[5] D.P. Ahlfeld, and A.E. Mulligan, *Optimal management of flow in groundwater systems* (Academic Press, San Diego, 2000).

[6] S.M. Gorelick, A review of distributed parameter groundwater management modeling methods, *Water Resour. Res.*, *19*(2), 1983, 305-319.

[7] R.A. Freeze, and S.M. Gorelick, Convergence of stochastic optimization and decision analysis in the engineering design of aquifer remediation, *Ground water*, *37*(6), 1999, 934-954.

[8] K.J. Larson, H. Basagaoglu, and M.A. Marino, Prediction of optimal safe groundwater yield and land subsidence in the Los Banos-Kettleman City Area, California, using a calibrated numerical simulation model, *Journal of Hydrology*, *242*(1-2), 2001, 79-102.

[9] S.P. Phillips, C.S. Carlson, L.F. Metzger, J.F. Howle, D.L. Galloway, M. Sneed, M.E. Ikehara, K.W. Hudnut, and N.E. King, *Analysis of tests of subsurface injection, storage, and recovery of freshwater in Lancaster, Antelope Valley, California* (U.S. Geological Survey Water-Resources Investigations Report 03-4061, 2003).

[10] T.L.Tsai, *The development and application of model of regional land subsidence due to groundwater overpumping* (PhD thesis, National Chiao Tung Univ., Hsinchu, Taiwan, 2001).

[11] Y.L. Chang, T.L. Tsai, J.C. Yang, and Y.K. Tung, Stochastically optimal groundwater management considering land subsidence, *J. Water Resour. Plan. Manage.*, *133*(6), 2007, 486-498.

[12] N.M. Safai, and G.F. Pinder, Vertical and horizontal land deformation due to fluid withdrawal, *International Journal for Numerical and Analytical Methods in Geomechanics*, *4*(2), 1980, 131-142.

[13] M.P. Anderson, and W.W. Woessner, *Applied groundwater modeling: simulation of flow and advective transport* (Academic Press, Inc., London, 1991).

[14] J. Bear, and A. Verruijt, *Modeling groundwater flow and pollution* (D. Reidel Publishing Company, Taiwan, 1987).

[15] S.A. Leake, Interbed storage changes and compaction in models of regional groundwater-flow, *Water Resour. Res.*, *26*(9), 1990, 1939-1950.

[16] D.C. Helm, One-dimensional simulation of aquifer-system compaction near Pixley, California, 1, Constant parameters, *Water Resour. Res.*, *11*, 1975, 465-478.

[17] M.S. Gutierrez, and R.W. Lewis, Coupling fluid flow and deformation in underground formations, *Journal of Engineering Mechanics*, *128*(7), 2002, 779-787.

[18] S.P. Uryas'ev, New variable-metric algorithms for nondifferentiable optimization problems, *Journal of Optimization Theory and Applications*, *71*(2), 1991, 359-388.

[19] Y.F. Yang, and H.L. Dong, A trust region algorithm for constrained nonsmooth optimization problems, *Journal of Computation Mathematics*, *19*(4), 2001, 357-364.

[20] G. Sierlsma, *Linear and integer program : theory and practice* (Dekker, New York, 1996).

[21] J.D. Bredehoeft, and G.F. Pinder, Digital analysis of areal flow in multiaquifer groundwater systems: a quasi three-dimensional model, *Water Resour. Res.*, *6*(3), 1970, 883-888.